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# Equations and Variable Definitions from Coal Mining Study

## Main Equations

### Equation (1) - Cobb-Douglas Production Function

$$q\_{ft}=β\_{l}l\_{ft}+β\_{m}m\_{ft}+β\_{k}k\_{ft}+ω\_{ft}$$

### Equation (2) - TFP Transition Process

$$ω\_{ft}=ρω\_{ft−1}+u\_{ft}$$

### Labor Supply Function - Equation (3)

$$W\_{it}^{l}=L\_{it}^{Ψ^{l}}η\_{it}$$

### Cost Minimization Objective - Equation (4)

$$\min\_{L\_{ft},M\_{ft}}\left\{\sum\_{g\in F\_{i\left(f\right)t}}^{​}\left(λ\_{fgt}\left(L\_{gt}W\_{gt}^{l}+M\_{gt}W\_{gt}^{m}\right)\right)−MC\_{ft}\left(Q\left(L\_{ft},M\_{ft},K\_{ft},Ω\_{ft};β\right)−Q\_{ft}\right)\right\}$$

### No Collusion Cost Minimization - Equation (5)

$$\min\_{L\_{ft},M\_{ft}}\left(L\_{ft}W\_{ft}^{l}+M\_{ft}W\_{ft}^{m}\right)−MC\_{ft}\left(Q\_{ft}−Q\left(L\_{ft},M\_{ft},K\_{ft},Ω\_{ft};β\right)\right)$$

### First-Order Condition for Labor (No Collusion) - Equation (6)

$$L\_{ft}\frac{∂W\_{it}^{l}}{∂L\_{it}}+W\_{it}^{l}=\frac{∂Q\_{ft}}{∂L\_{ft}}\frac{P\_{ft}}{μ\_{ft}}$$

### Markdown Under No Collusion - Equation (7)

$$\tilde{m}\_{ft}^{l}=1+s\_{ft}^{l}Ψ^{l}$$

### Perfect Collusion Cost Minimization - Equation (8)

$$\min\_{L\_{ft},M\_{ft}}\left\{\sum\_{g\in F\_{i\left(f\right)t}}^{​}\left(L\_{gt}W\_{gt}^{l}+M\_{gt}W\_{gt}^{m}\right)−MC\_{ft}\left(Q\_{ft}−Q\left(L\_{ft},M\_{ft},K\_{ft},Ω\_{ft};β\right)\right)\right\}$$

### First-Order Condition for Labor (Perfect Collusion) - Equation (9)

$$L\_{it}\frac{∂W\_{it}^{l}}{∂L\_{it}}+W\_{it}^{l}=\frac{∂Q\_{ft}}{∂L\_{ft}}\frac{P\_{ft}}{μ\_{ft}}$$

### Markdown Under Perfect Collusion - Equation (10)

$$\overline{m}\_{ft}^{l}=1+Ψ^{l}$$

### General First-Order Condition - Equation (11)

$$W\_{it}^{l}+\tilde{λ}\_{ft}\frac{∂W\_{it}^{l}}{∂L\_{it}}L\_{it}=\frac{∂Q\_{ft}}{∂L\_{ft}}\frac{P\_{ft}}{μ\_{ft}}$$

### General Markdown Expression - Equation (12)

$$m\_{ft}^{l}=1+\tilde{λ}\_{ft}Ψ^{l}$$

### Markup from Materials - Equation (13)

$$μ\_{ft}=\frac{β\_{m}}{α\_{ft}^{m}}$$

### Core Identification Equation - Equation (14)

$$m\_{ft}^{l}=1+\tilde{λ}\_{ft}Ψ^{l}=\frac{β\_{l}α\_{ft}^{m}}{β\_{m}α\_{ft}^{l}}$$

### Conduct Parameter Definition - Equation (15)

$$\hat{λ}\_{ft}=\frac{m\_{ft}^{l}−\tilde{m}\_{ft}^{l}}{\overline{m}\_{ft}^{l}−\tilde{m}\_{ft}^{l}}$$

### Moment Conditions for Production Function - Equation (16)

$$E\left[u\_{ft}|\left(l\_{fr−1},m\_{fr−1},k\_{fr},w\_{r−1}^{agri}\right)\_{r\in \{2,...,t\}}\right]=0$$

### GMM Moment Conditions - Equation (17)

$$E\left[q\_{ft}−ρq\_{ft−1}−β\_{0}\left(1−ρ\right)−β\_{l}\left(l\_{ft}−ρl\_{ft−1}\right)−β\_{m}\left(m\_{ft}−ρm\_{ft−1}\right)−β\_{k}\left(k\_{ft}−ρk\_{ft−1}\right)|\left(l\_{ft−1},m\_{ft−1},k\_{ft},k\_{ft−1},w\_{t−1}^{agri}\right)\right]=0$$

## Additional Key Definitions

### Markdown Definition

$$m\_{ft}^{l}≡\frac{MRPL\_{ft}}{W\_{ft}^{l}}$$

### Percentage Markdown Definition

$$d\_{ft}^{l}≡\frac{MRPL\_{ft}−W\_{ft}^{l}}{MRPL\_{ft}}=\frac{m\_{ft}^{l}−1}{m\_{ft}^{l}}$$

### Markup Definition

$$μ\_{ft}≡\frac{P\_{ft}}{MC\_{ft}}$$

### Labor Market Share

$$s\_{ft}^{l}≡\frac{L\_{ft}}{L\_{it}}$$

### Revenue Share of Labor

$$α\_{ft}^{l}≡\frac{W\_{ft}^{l}L\_{ft}}{P\_{ft}Q\_{ft}}$$

### Revenue Share of Materials

$$α\_{ft}^{m}≡\frac{W\_{ft}^{m}M\_{ft}}{P\_{ft}Q\_{ft}}$$

### Inverse Labor Supply Elasticity (Firm-level)

$$θ\_{ft}^{l}≡\frac{∂W\_{ft}^{l}}{∂L\_{ft}}\frac{L\_{ft}}{W\_{ft}^{l}}$$

### Inverse Materials Supply Elasticity (Firm-level)

$$θ\_{ft}^{m}≡\frac{∂W\_{ft}^{m}}{∂M\_{ft}}\frac{M\_{ft}}{W\_{ft}^{m}}$$

## Variable Definitions

| Variable | Definition |
| --- | --- |
| $Q\_{ft}$ | Output (tonnage of coal) extracted by firm $f$ in year $t$ |
| $L\_{ft}$ | Amount of effective labor throughout the year for firm $f$ in year $t$ |
| $M\_{ft}$ | Amount of intermediate inputs purchased by firm $f$ in year $t$ |
| $K\_{ft}$ | Capital stock (steam engines) used by firm $f$ in year $t$ |
| $q\_{ft}$ | Logarithm of output $Q\_{ft}$ |
| $l\_{ft}$ | Logarithm of labor $L\_{ft}$ |
| $m\_{ft}$ | Logarithm of materials $M\_{ft}$ |
| $k\_{ft}$ | Logarithm of capital $K\_{ft}$ |
| $β\_{l}$ | Output elasticity of labor |
| $β\_{m}$ | Output elasticity of materials |
| $β\_{k}$ | Output elasticity of capital |
| $ω\_{ft}$ | Log total factor productivity |
| $u\_{ft}$ | Unexpected productivity shock |
| $ρ$ | Serial correlation parameter in productivity process |
| $W\_{it}^{l}$ | Wage in labor market $i$ in year $t$ |
| $W\_{ft}^{l}$ | Wage for firm $f$ in year $t$ |
| $W\_{ft}^{m}$ | Price of materials for firm $f$ in year $t$ |
| $L\_{it}$ | Market-level employment in market $i$ in year $t$ |
| $Ψ^{l}$ | Inverse market-level labor supply elasticity |
| $η\_{it}$ | Market-specific residual in labor supply |
| $λ\_{fgt}$ | Collusion weight that firm $f$ puts on firm $g$’s costs |
| $\tilde{λ}\_{ft}$ | Conduct parameter (firm-level aggregate of bilateral conduct parameters) |
| $\hat{λ}\_{ft}$ | Normalized conduct parameter ranging from 0 to 1 |
| $MC\_{ft}$ | Marginal cost for firm $f$ in year $t$ |
| $P\_{ft}$ | Coal price for firm $f$ in year $t$ |
| $MRPL\_{ft}$ | Marginal revenue product of labor for firm $f$ in year $t$ |
| $m\_{ft}^{l}$ | Wage markdown (ratio of MRPL to wage) |
| $\tilde{m}\_{ft}^{l}$ | Wage markdown under no collusion |
| $\overline{m}\_{ft}^{l}$ | Wage markdown under perfect collusion |
| $d\_{ft}^{l}$ | Percentage wage markdown |
| $μ\_{ft}$ | Price markup (ratio of price to marginal cost) |
| $s\_{ft}^{l}$ | Labor market share of firm $f$ |
| $α\_{ft}^{l}$ | Revenue share of labor for firm $f$ in year $t$ |
| $α\_{ft}^{m}$ | Revenue share of materials for firm $f$ in year $t$ |
| $θ\_{ft}^{l}$ | Inverse firm-level labor supply elasticity |
| $θ\_{ft}^{m}$ | Inverse firm-level materials supply elasticity |
| $F\_{i\left(f\right)t}$ | Set of firms in market $i$ (where firm $f$ operates) in year $t$ |
| $w\_{t−1}^{agri}$ | Agricultural wages in Belgium in year $t−1$ (instrument) |

## Tab-Separated Variable Definitions

Variable Definition
Q\_ft Output (tonnage of coal) extracted by firm f in year t
L\_ft Amount of effective labor throughout the year for firm f in year t
M\_ft Amount of intermediate inputs purchased by firm f in year t
K\_ft Capital stock (steam engines) used by firm f in year t
q\_ft Logarithm of output Q\_ft
l\_ft Logarithm of labor L\_ft
m\_ft Logarithm of materials M\_ft
k\_ft Logarithm of capital K\_ft
beta\_l Output elasticity of labor
beta\_m Output elasticity of materials
beta\_k Output elasticity of capital
omega\_ft Log total factor productivity
u\_ft Unexpected productivity shock
rho Serial correlation parameter in productivity process
W\_l\_it Wage in labor market i in year t
W\_l\_ft Wage for firm f in year t
W\_m\_ft Price of materials for firm f in year t
L\_it Market-level employment in market i in year t
Psi\_l Inverse market-level labor supply elasticity
eta\_it Market-specific residual in labor supply
lambda\_fgt Collusion weight that firm f puts on firm g's costs
lambda\_tilde\_ft Conduct parameter (firm-level aggregate of bilateral conduct parameters)
lambda\_hat\_ft Normalized conduct parameter ranging from 0 to 1
MC\_ft Marginal cost for firm f in year t
P\_ft Coal price for firm f in year t
MRPL\_ft Marginal revenue product of labor for firm f in year t
m\_l\_ft Wage markdown (ratio of MRPL to wage)
m\_l\_tilde\_ft Wage markdown under no collusion
m\_l\_bar\_ft Wage markdown under perfect collusion
d\_l\_ft Percentage wage markdown
mu\_ft Price markup (ratio of price to marginal cost)
s\_l\_ft Labor market share of firm f
alpha\_l\_ft Revenue share of labor for firm f in year t
alpha\_m\_ft Revenue share of materials for firm f in year t
theta\_l\_ft Inverse firm-level labor supply elasticity
theta\_m\_ft Inverse firm-level materials supply elasticity
F\_i\_f\_t Set of firms in market i (where firm f operates) in year t
w\_agri\_t\_minus\_1 Agricultural wages in Belgium in year t-1 (instrument)

# Table 1

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Table 1: TABLE 1: Model EstimatesTABLE 1: Model Estimates

|  | log(Output) | log(Output) | log(Output) |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| A. Production Function, log(Output) |  |  |  |
| log(Labor) *bl* | .794 | .699 | .661 |  |  |  |
|  | (.034) | (.327) | (.041) |  |  |  |
| log(Materials) *bm* | .275 | .222 | .237 |  |  |  |
|  | (.028) | (.138) | (.080) |  |  |  |
| log(Capital) *bk* | 2.008 | .153 | .102 |  |  |  |
|  | (.140) | (.075) | (.088) |  |  |  |
| Serial correlation TFP *r* | .866 | .853 |  |  |  |  |
|  | (.198) | (.157) |  |  |  |  |
| Method | OLS | GMM | GMM |  |  |  |
| RTS | Free | Free | Fixed at 1.05 |  |  |  |
| *R*2 | .941 | .938 | .826 |  |  |  |
| Hansen *J*-test |  | 2.34 | 2.72 |  |  |  |
| Hansen *J*-test *p*-value |  | .126 | .255 |  |  |  |
| Number of firms | 166 | 159 | 159 |  |  |  |
| Observations | 4,480 | 4,005 | 4,005 |  |  |  |
| B. Markdowns and Markups |  |  |  |
| Median markdown | 1.541 | 1.680 | 1.486 |  |  |  |
|  | (.193) | (.450) | (.330) |  |  |  |
| Average markdown | 1.676 | 1.828 | 1.616 |  |  |  |
|  | (.224) | (.491) | (.361) |  |  |  |
| Median markup | .884 | .714 | .763 |  |  |  |
|  | (.112) | (.494) | (.287) |  |  |  |
| Average markup | .946 | .764 | .816 |  |  |  |
|  | (.120) | (.535) | (.315) |  |  |  |
| Method | OLS | GMM | GMM |  |  |  |
| RTS | Free | Free | Fixed at 1.05 |  |  |  |
| C. Labor Supply |
|  | log(Wage) |  | log(Wage) |  |  |  |
|  | Est. | SE | Est. | SE |  |  |
| log(Employment) | .066 | .006 | 1.009 | .265 |  |  |
| Method | OLS |  | IV |  |  |  |
| First-stage *F*-statistic |  |  | 462 |  |  |  |
| Hansen *J*-test |  |  | 5.92 |  |  |  |
| Hansen *J*-test *p*-value |  |  | .014 |  |  |  |
| Observations | 1,990 |  | 1,990 |  |  |  |
| Firm-level elasticity | 155.56 |  | 10.172 |  |  |  |

 |

## Derivation of (14)

Looking at equation (14) and the context provided, I can explain how the wage markdown is derived by combining the two markup estimates from the production approach.

### Cost minimization

From the cost minimization problem, the first-order condition for labor (equation 11) is:

$$W\_{it}^{l}+\tilde{λ}\_{ft}⋅\frac{∂W\_{it}^{l}}{∂L\_{it}}⋅L\_{it}=\frac{∂Q\_{ft}}{∂L\_{ft}}⋅\frac{P\_{ft}}{μ\_{ft}}$$

Where the right side is the marginal revenue product of labor (MRPL).

Rearranging and using: - $β\_{l}$ = output elasticity of labor - $α\_{ft}^{l}=\frac{W\_{ft}^{l}⋅L\_{ft}}{P\_{ft}⋅Q\_{ft}}$ = revenue share of labor

The markup derived from the labor first-order condition becomes:

$$μ\_{ft}=\frac{β\_{l}}{α\_{ft}^{l}⋅\left(1+\tilde{λ}\_{ft}⋅Ψ^{l}\right)}$$

From (11)

$$W\_{it}^{l}+\tilde{λ}\_{ft}\frac{∂W\_{it}^{l}}{∂L\_{it}}L\_{it}=\frac{∂Q\_{ft}}{∂L\_{ft}}\frac{P\_{ft}}{μ\_{ft}}$$

From the Cobb-Douglas production function: $Q\_{ft}=e^{β\_{l}l\_{ft}+β\_{m}m\_{ft}+β\_{k}k\_{ft}+ω\_{ft}}$

Taking the derivative with respect to labor:

$$\frac{∂Q\_{ft}}{∂L\_{ft}}=\frac{∂Q\_{ft}}{∂l\_{ft}}⋅\frac{∂l\_{ft}}{∂L\_{ft}}=β\_{l}⋅Q\_{ft}⋅\frac{1}{L\_{ft}}=β\_{l}\frac{Q\_{ft}}{L\_{ft}}$$

Substitute this into the RHS of equation (11)

$$\frac{∂Q\_{ft}}{∂L\_{ft}}\frac{P\_{ft}}{μ\_{ft}}=β\_{l}\frac{Q\_{ft}}{L\_{ft}}\frac{P\_{ft}}{μ\_{ft}}=β\_{l}\frac{P\_{ft}Q\_{ft}}{μ\_{ft}L\_{ft}}$$

Multiply both sides by $L\_{ft}$

$$L\_{ft}W\_{it}^{l}+\tilde{λ}\_{ft}\frac{∂W\_{it}^{l}}{∂L\_{it}}L\_{it}^{2}=β\_{l}\frac{P\_{ft}Q\_{ft}}{μ\_{ft}}$$

From equation (3): $W\_{it}^{l}=L\_{it}^{Ψ^{l}}η\_{it}$

Taking the derivative:

$$\frac{∂W\_{it}^{l}}{∂L\_{it}}=Ψ^{l}L\_{it}^{Ψ^{l}−1}η\_{it}=Ψ^{l}\frac{W\_{it}^{l}}{L\_{it}}$$

Substitute this back to the above

$$L\_{ft}W\_{it}^{l}+\tilde{λ}\_{ft}Ψ^{l}\frac{W\_{it}^{l}}{L\_{it}}L\_{it}^{2}=β\_{l}\frac{P\_{ft}Q\_{ft}}{μ\_{ft}}$$

$$L\_{ft}W\_{it}^{l}+\tilde{λ}\_{ft}Ψ^{l}W\_{it}^{l}L\_{it}=β\_{l}\frac{P\_{ft}Q\_{ft}}{μ\_{ft}}$$

$$W\_{it}^{l}L\_{ft}\left(1+\tilde{λ}\_{ft}Ψ^{l}\right)=β\_{l}\frac{P\_{ft}Q\_{ft}}{μ\_{ft}}$$

Markup

$$μ\_{ft}=\frac{β\_{l}P\_{ft}Q\_{ft}}{W\_{it}^{l}L\_{ft}\left(1+\tilde{λ}\_{ft}Ψ^{l}\right)}$$

Define the revenue share of labor: $α\_{ft}^{l}=\frac{W\_{ft}^{l}L\_{ft}}{P\_{ft}Q\_{ft}}$, therefore: $W\_{ft}^{l}L\_{ft}=α\_{ft}^{l}P\_{ft}Q\_{ft}$

$$μ\_{ft}=\frac{β\_{l}P\_{ft}Q\_{ft}}{α\_{ft}^{l}P\_{ft}Q\_{ft}\left(1+\tilde{λ}\_{ft}Ψ^{l}\right)}=\frac{β\_{l}}{α\_{ft}^{l}\left(1+\tilde{λ}\_{ft}Ψ^{l}\right)}$$

This gives us the markup expression derived from the labor first-order condition, which incorporates the wage markdown term $\left(1+\tilde{λ}\_{ft}Ψ^{l}\right)$ in the denominator.

### Markup from materials (following De Loecker & Warzynski 2012)

For materials, since firms are price-takers (competitive input market), the standard markup formula applies:

$$μ\_{ft}=\frac{β\_{m}}{α\_{ft}^{m}}$$

Where $α\_{ft}^{m}$ is the revenue share of materials.

From the cost minimization problem, the first-order condition for materials is:

$$W\_{ft}^{m}=\frac{∂Q\_{ft}}{∂M\_{ft}}\frac{P\_{ft}}{μ\_{ft}}$$

From the Cobb-Douglas production function: $Q\_{ft}=e^{β\_{l}l\_{ft}+β\_{m}m\_{ft}+β\_{k}k\_{ft}+ω\_{ft}}$

Taking the derivative with respect to materials:

$$\frac{∂Q\_{ft}}{∂M\_{ft}}=\frac{∂Q\_{ft}}{∂m\_{ft}}⋅\frac{∂m\_{ft}}{∂M\_{ft}}=β\_{m}⋅Q\_{ft}⋅\frac{1}{M\_{ft}}=β\_{m}\frac{Q\_{ft}}{M\_{ft}}$$

Substitute this into the RHS

$$W\_{ft}^{m}=β\_{m}\frac{Q\_{ft}}{M\_{ft}}\frac{P\_{ft}}{μ\_{ft}}=β\_{m}\frac{P\_{ft}Q\_{ft}}{μ\_{ft}M\_{ft}}$$

$$W\_{ft}^{m}M\_{ft}=β\_{m}\frac{P\_{ft}Q\_{ft}}{μ\_{ft}}$$

Solve for the markup

$$μ\_{ft}=\frac{β\_{m}P\_{ft}Q\_{ft}}{W\_{ft}^{m}M\_{ft}}$$

Define the revenue share of materials: $α\_{ft}^{m}=\frac{W\_{ft}^{m}M\_{ft}}{P\_{ft}Q\_{ft}}$

Therefore: $W\_{ft}^{m}M\_{ft}=α\_{ft}^{m}P\_{ft}Q\_{ft}$

$$μ\_{ft}=\frac{β\_{m}P\_{ft}Q\_{ft}}{α\_{ft}^{m}P\_{ft}Q\_{ft}}=\frac{β\_{m}}{α\_{ft}^{m}}$$

This gives us the standard De Loecker & Warzynski (2012) markup formula:

$$μ\_{ft}=\frac{β\_{m}}{α\_{ft}^{m}}$$

### Equate the two markup expressions

Since both expressions equal $μ\_{ft}$:

$$\frac{β\_{l}}{α\_{ft}^{l}⋅\left(1+\tilde{λ}\_{ft}⋅Ψ^{l}\right)}=\frac{β\_{m}}{α\_{ft}^{m}}$$

Rearranging:

$$1+\tilde{λ}\_{ft}⋅Ψ^{l}=\frac{β\_{l}⋅α\_{ft}^{m}}{β\_{m}⋅α\_{ft}^{l}}$$

From equation (12), we know that:

$$m\_{ft}^{l}=1+\tilde{λ}\_{ft}⋅Ψ^{l}$$

Therefore:

$$m\_{ft}^{l}=\frac{β\_{l}⋅α\_{ft}^{m}}{β\_{m}⋅α\_{ft}^{l}}$$

This is equation (14) in the paper.

The wage markdown $m\_{ft}^{l}$ can be calculated in two equivalent ways:

1. **Labor supply approach**: $1+\tilde{λ}\_{ft}⋅Ψ^{l}$ (depends on conduct parameter and labor supply elasticity)
2. **Production approach**: $\frac{β\_{l}⋅α\_{ft}^{m}}{β\_{m}⋅α\_{ft}^{l}}$ (depends only on production function parameters and cost shares)